A Decentralized Decision Making System for Provision of Public Goods

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Abstract. First, under the perfect information condition, a decentralized decision making system for public goods provision is presented. In this system, each individual decides how to use a certain amount of the public budget. That is, each individual is given the disposal right of a certain amount of the public budget, and can decide how to use the amount he is given. Second, this paper shows that the allocation of resources in the equilibrium of this decentralized system is efficient in a sense.

1. Introduction

The allocation of resources in an economy with public goods would not be efficient automatically in a market mechanism. Public goods are supplied through political processes in the real economy. But, the allocation decided through political processes would not be efficient, too. Under the view that politicians seek to win elections and bureaucrats seek to maximize their budget, the provision of public goods through political processes therefore must be inefficient.

Under the condition that both the market mechanism and political processes do not provide public goods efficiently, how can the allocation efficiency be improved ? The reason why the market mechanism fails to provide public goods results from the nature of the public goods themselves, non-excludability and non-rivalness. On the other hand, the failure to provide public goods through the political processes does not come from the nature of public goods, but partly from the collective decision making system.

To analyze the allocation of resources with public goods, usually

Samuelson's condition is used[i]. Samuelson's condition is concerned with the relationship both between public goods and private goods and among public goods themselves. If Samuelson's condition is satisfied for all the public goods, the allocation of resources is Pareto efficient. But, to adjust the allocation, the demand revelation problem must be solved. Although many attempts such as Lindahl (1967), Bowen (1943) and Clarke (1971) have been made to solve the problem, a practical solution has never been found.[ii] To find a practical solution, not only the relationship between public goods and private goods is important for efficiency, but also the relationship among the public goods themselves.

In this paper, while the relationship between public goods and private goods is not analyzed, the relationships among public goods are considered. That is, under the condition that the resources used for public goods have been decided already, a system in which an efficient provision of each public good is generated is presented.

2. The Model

There are n individuals in a society. The budget system is composed of two parts. One is the total level of expenditures decided by tax, the other is the disposal right of the budget.[iii] Tax is collected from individuals for public goods provision. The amount of each individual's tax payment and the total amount of the budget are decided by the tax structure the society adopts.[iv] In most of the congressional representative democracies, the disposal rights of public budgets are given to the congress or to the representatives. But in this model, it is assumed that all individuals have a part of disposal right of the public budget. Some parts of Disposal rights of the public budget are distributed to individuals. Apart from the tax structure, the distribution of these disposal rights is given.[v]

The amount of the public budget that the i-th individual can decide how to use is denoted by T_1. There are 2 kinds of public goods: y_1 supplied by bureau 1, y_2 supplied by bureau 2. In the system suggested in this paper, each individual decides a certain amount of the public budget. That is, while the amount of one's tax payment is decided by the tax structure, one can select the method for a part of the public budget. The i-th individual decides how much to contribute to each type of public goods from his disposable amount, T_i. When the contributions to y_1 are t_i^1, the contributions to y_2 become $t_i^2 = T_i - t_i^1$. The i-th individual decides t_i^1 to maximize the utility.

For example, there are 2 individuals, A and B, and 2 kinds of public goods, national defense and roads, in society. Assuming that the disposable amount of A is 1 billion yen, and that of B is .5 billion yen, if A decides to contribute .3 billion yen to national defense and .7 billion yen to roads, and if B decides to contribute .4 billion yen to national defense and .1 billion yen to roads, then the budget for national defense becomes .7 billion yen, the sum of .3 billion yen contributed by A and .4 billion yen contributed by B. In the same way the budget for roads becomes .8 billion yen.

The equilibrium

Each individual decides his contributions to each type of public goods to maximize his utility under the perfect information condition.

The disposable amount of the i-th individual is T_1, the contribution to y_1 is denoted by t_i^1. There are n individuals and 2 public goods. The relation of one's disposable amount and the contributions to public goods is now,

$$T_1 = t_i^1 + t_i^2 \quad i = 1, 2, ..., n \text{ ....(2.1)}$$

with $t_i^1 \geq 0$ for $i = 1, 2, ..., n \quad j = 1, 2.$

The utility of the i-th individual denoted by U_i depends on the amount of consumption of private goods denoted by X and that of the j-th public goods denoted by Y_j. And the utility function of the i-th individual is assumed additively separable. So the utility function is,

$$U_i = f_i(X) + g_i(Y_1, Y_2) \text{ ....(2.2).}$$

For simplicity, the part of the utility function relating to public goods is assumed to be linear. Then it is[vi],

$$g_i(Y_1, Y_2) = a_i Y_1 + b_i Y_2 \text{ ....(2.3).}$$

The quantity of each type of public goods is decided by the sum of the contributions from each individual and the average cost. So it is,

$$Y_j = \frac{\sum_{i=1}^{n} t_i^j}{c_j} \text{ ....(2.4).}$$

where c_j is the average cost of providing the j-th public goods.

Each individual decides the allocation of the disposable amount under these conditions. That is, the i-th individual solves the next utility maximization problem, given the average cost of each of the public goods and the behavior of the other individuals.

$$\underset{t_i^1}{Max} U_i \quad s.t. T_i = t_i^1 + t_i^2 \quad Y_j = \frac{\sum_{i=1}^{n} t_i^j}{c_j} \quad i = 1, 2, ..., n \quad j = 1, 2 \text{..................(A)}$$

Maximizing U_i with respect to t_i^1 yields,

$$\frac{\partial U_i}{\partial t_i^1} = \frac{a_i}{c_1} - \frac{b_i}{c_2} \text{ ....(2.5).}$$

Then,

(1) When $\frac{a_i}{b_i} > \frac{c_1}{c_2}$ that is $\frac{\partial U_i}{\partial t_i^1} > 0$, the utility of the i-th individual increases with more spending to y_1. So, the i-th individual allocates the entire amount to y_1. Then, $t_i^1 = T_i$.

(2) When $\frac{a_i}{b_i} < \frac{c_1}{c_2}$ that is $\frac{\partial U_i}{\partial t_i^1} < 0$, the utility of the i-th individual increases with less spending to y_1. So, the i-th individual allocates the entire amount to y_2. Then, $t_i^1 = 0$.

(3) When $\frac{a_i}{b_i} = \frac{c_1}{c_2}$ that is $\frac{\partial U_i}{\partial t_i^1} = 0$, because y_1 and y_2 are indifferent to the i-th individual, all allocations are possible. So, $0 \le t_i^1 \le T_i$.[vii]

That is,

$$\begin{cases} t_i^{1*} = T_i & if \ \frac{a_i}{b_i} > \frac{c_1}{c_2} \\ 0 \le t_i^{1*} \le T_i & if \ \frac{a_i}{b_i} = \frac{c_1}{c_2} \\ t_i^{1*} = 0 & if \ \frac{a_i}{b_i} < \frac{c_1}{c_2} \end{cases} \text{ ....(2.6).}$$

Theorem 1 : The set of strategies $N^* = (t_1^{1*}, t_1^{2*}, ..., t_i^{1*}, t_i^{2*}, ..., t_n^{1*}, t_n^{2*})$ that satisfies (2.6) is a Nash equilibrium.

Proof:

$N^* = (t_1^{1*}, t_1^{2*}, ..., t_i^{1*}, t_i^{2*}, ..., t_n^{1*}, t_n^{2*})$ is the set of strategies which satisfies (2.6), $N' = (t_1^{1*}, t_1^{2*}, ..., t_i^{1'}, t_i^{2'}, ..., t_n^{1*}, t_n^{2*})$ is the set of strategies when only the i-th individual's strategies do not satisfy (2.6). The utility of the i-th individual under N^* and N' is, from (2.2), (2.3), respectively,

$$\begin{aligned} U_i(N^*) &= f_i(X_i) + a_i\left(\frac{\sum_{i=1}^n t_i^{1^*}}{c_1}\right) + b_i\left(\frac{\sum_{i=1}^n t_i^{2^*}}{c_2}\right) \\ &= f_i(X_i) + a_i\left(\frac{\sum_{i=1}^n t_i^{1^*}}{c_1}\right) + b_i\left(\frac{T - \sum_{i=1}^n t_i^{1^*}}{c_2}\right) \quad \text{....................(2.7)} \\ &= f_i(X_i) + a_i\left(\frac{\sum_{j\neq i} t_j^{1^*}}{c_1} + \frac{t_i^{1^*}}{c_1}\right) + b_i\left(\frac{T - \sum_{j\neq i} t_j^{1^*}}{c_2} - \frac{t_i^{1^*}}{c_2}\right) \end{aligned}$$

$$\begin{aligned} U_i(N') &= f_i(X_i) + a_i\left(\frac{\sum_{j\neq i} t_j^{1^*} + t_i^{1'}}{c_1}\right) + b_i\left(\frac{\sum_{j\neq i} t_j^{2^*} + t_i^{2'}}{c_2}\right) \\ &= f_i(X_i) + a_i\left(\frac{\sum_{j\neq i} t_j^{1^*}}{c_1} + \frac{t_i^{1'}}{c_1}\right) + b_i\left(\frac{T - \sum_{j\neq i} t_j^{1^*}}{c_2} - \frac{t_i^{1'}}{c_2}\right) \end{aligned} \quad \text{.....................(2.8).}$$

Then,

$$\begin{aligned} U_i(N^*) - U_i(N') &= \frac{a_i}{c_1}\left(t_i^{1*} - t_i^{1'}\right) + \frac{b_i}{c_2}\left(t_i^{1*} - t_i^{1'}\right) \\ &= \left(t_i^{1*} - t_i^{1'}\right)\left(\frac{a_i}{c_1} - \frac{b_i}{c_2}\right) \end{aligned} \quad \text{...................(2.9).}$$

(i) When $\frac{a_i}{c_1} - \frac{b_i}{c_2} > 0$ that is $\frac{a_i}{b_i} > \frac{c_1}{c_2}$, from (2.6), $t_i^{1*} = T_i$. And $0 < t_i^{1'} < T_i$. Then, $U_i(N^*) - U_i(N') = \left(T_i - t_i^{1'}\right)\left(\frac{a_i}{c_1} - \frac{b_i}{c_2}\right) > 0$

...................(2.10).

(ii) When $\frac{a_i}{c_1} - \frac{b_i}{c_2} < 0$ that is $\frac{a_i}{b_i} < \frac{c_1}{c_2}$, from (2.1.6), $t_i^{1*} = 0$. And

$$0 < t_i^{1'} < T_i. \quad \text{Then, } U_i(N^*) - U_i(N') = -t_i^{1'}\left(\frac{a_i}{c_1} - \frac{b_i}{c_2}\right) > 0$$

....................(2.11).

(iii) When $\frac{a_i}{c_1} - \frac{b_i}{c_2} = 0$ that is $\frac{a_i}{b_i} = \frac{c_1}{c_2}$, for all t_i^{1*},

$$U_i(N^*) - U_i(N') = \left(\frac{T_i}{2} - t_i^{1'}\right)\left(\frac{a_i}{c_1} - \frac{b_i}{c_2}\right) = 0 \quad ....(2.12).$$

Q.E.D.

3. Allocation of Resources

In this decentralized decision making system of public goods provision, what characteristics does this allocation of resources have ? In this model, the ratio of resources used for public goods is treated as given[viii], because the amount of tax levied on each individual is exogenous. So, the allocation of resources in this model does not satisfy Samuelson's condition.

Given the ratio of the resources for private goods and that for public goods, that is, the resources used for public goods have been decided already, it is important to analyze the amount of each public good provided. The problem is to determine the efficient expenditure schedule of public goods under a budget.

A budget system generally consists of two elements, the total level of expenditures and the distribution of the disposal right of the governmental income. Under a budget system, if it is impossible to change the expenditure schedule without any decrease in anyone's utilities, the expenditure schedule can be said to be efficient. This efficiency is defined below as The Allocation Efficiency Subjected to Budget System. In this context, a budget system contains the overall level of governmental activity that is decided by a tax system, and the distribution of the disposition of resources that the government has. The overall level of governmental activity means the quantity of the resources that the government withdraws from the private sector.

Definition: The Allocation Efficiency Subjected to Budget System

Given a budget system[ix], if there is no such set of consumptions of private goods and public goods as $(X'_1,...,X'_i,...,X'_n,Y'_1,Y'_1)$ under which the utility of the i-th individual is higher than under $(X^*_1,...,X^*_i,...,X^*_n,Y^*_1,Y^*_1)$ for any i-th individual, $(X^*_1,...,X^*_i,...,X^*_n,Y^*_1,Y^*_1)$ is efficient as regards the meaning of the Allocation Efficiency Subjected to Budget System.

Theorem 2: If the utility function of each individual is linier and additively separable, and if the private goods market is the perfect competitive market, the allocation in the equilibrium of this model, $(X^*_1,...,X^*_i,...,X^*_n,Y^*_1,Y^*_2)$, is efficient as regards the meaning of the Allocation Efficiency Subjected to Budget System.

Proof:

Assume,

$$Y_1^* = \frac{R_1^*}{c_1},\; Y_2^* = \frac{R_2^*}{c_2} = \frac{T - R_1^*}{c_2},\; Y_1^{'} = \frac{R_1^* + e}{c_1},\; Y_2^{'} = \frac{T - R_1^* - e}{c_2} \quad \text{............(3.1)}$$

with $0 \leq R_1^* + e \leq T$ and $e = cons.$, where Y_1^* and Y_2^* are the quantities of public goods in the equilibrium.

Because, the utility function of the i-th individual is additively separable, the utility function of the i-th individual can be assumed as,

$U_i = f_i(X) + g_i(Y_1, Y_2)$.

Where X is a set of private goods. And the private goods market is assumed to be perfect competitive, X^* is a set of private goods at the equilibrium, X' is a set of private goods other than X^*. Thus,

$f(X_i^*) - f(X_i^{'}) > 0$ ....(3.2).

Now, compare the part of the utility function related to public goods,

$$g_i(Y_1^*, Y_2^*) - g_i(Y_1^{'}, Y_2^{'}) = a_1\frac{R_1^*}{c_1} + b_1\frac{T - R_1^*}{c_1} - a_1\frac{R_1^* + e}{c_1} - b_1\frac{T - R_1^* - e}{c_1}$$

$$= -e\left(\frac{a_i}{c_1} - \frac{b_i}{c_2}\right) \quad \text{..................(3.3).}$$

Then,

(i) When $\frac{a_i}{c_1} - \frac{b_i}{c_2} \geq 0$ that is $\frac{a_i}{b_i} \geq \frac{c_1}{c_2}$ for each i-th individual, from (2.6), for each i-th $t_i^{1*} = T_i$ and then $R_1^* = \Sigma_{i=1}^{n} T_i = T$. And, for $e \leq 0$,

$$g_i(Y_1^*, Y_2^*) - g_i(Y_1', Y_2') = -e\left(\frac{a_i}{c_1} - \frac{b_i}{c_2}\right) \geq 0 \text{ ....(3.4).}$$

(ii) When $\frac{a_i}{c_1} - \frac{b_i}{c_2} \leq 0$ that is $\frac{a_i}{b_i} \leq \frac{c_1}{c_2}$ for each i-th individual, from (2.6), for each i-th $t_i^{1*} = 0$ and then $R_1^* = 0$. And, for $0 \leq e \leq$ T,

$$g_i(Y_1^*, Y_2^*) - g_i(Y_1', Y_2') = -e\left(\frac{a_i}{c_1} - \frac{b_i}{c_2}\right) \geq 0 \text{ ....(3.5).}$$

(iii) When $\frac{a_i}{c_1} - \frac{b_i}{c_2} > 0$ that is $\frac{a_i}{b_i} > \frac{c_1}{c_2}$ for the i-th individual, and $\frac{a_k}{c_1} - \frac{b_k}{c_2} < 0$ that is $\frac{a_k}{b_k} > \frac{c_1}{c_2}$ for another k-th individual, if there is such a set as (Y_1', Y_2'), which gives higher utility to the i-th individual than (Y_1^*, Y_2^*),

$$g_i(Y_1^*, Y_2^*) - g_i(Y_1', Y_2') = -e\left(\frac{a_i}{c_1} - \frac{b_i}{c_2}\right) < 0$$

holds. For the i-th individual $\frac{a_i}{c_1} - \frac{b_i}{c_2} > 0$, and for $e > 0$.
But, when $e > 0$, the utility of the k-th individual is

$$g_k(Y_1^*, Y_2^*) - g_k(Y_1', Y_2') = -e\left(\frac{a_k}{c_1} - \frac{b_k}{c_2}\right) > 0.$$

Thus, there is no such e as it increases the utilities of all individuals simultaneously.

That is, $(X_1^*, \ldots, X_i^*, \ldots, X_n^*, Y_1^*, Y_2^*)$ satisfies the Allocation Efficiency Subjected to Budget System.

Q.E.D

4. Side transfer

In this model, side transfers of the private good or the public goods budget among individuals are not concerned. If such transfers are realized, because of the imperfect separation of the private goods market and the public goods market, may the free rider problem again become an issue ? No, the free rider does not become a problem in this system.

If individual A buys some disposal rights which individual B has, the utilities of both A and B improves. Individual A gets more from public goods than private goods, and individual B gets more from private goods than public goods.

5. Discussions

This paper presents an idea of decision making system for the provision of public goods. Because this model assumes perfect information, it may attract some criticism such that because it is impossible for each individual to know the detail of the public budget, this kind of decentralized system will not work[x].

It is necessary to analyze what happens when perfect information is lacking. Especially, the lack of information concerning others' utility will force some modification on this decentralized system. Under the perfect information condition, each individual knows the character of each goods and the others' payoff, utility. Of course, it is important to analyze both the character of each good and the others' utility. The lack of perfect information about the character of each good is as important as it is in the analysis of the private goods market, as mentioned above. More important is the information about others' utility. In the private goods market, such information is unnecessary. But, in this model, each decides given the others' choice. If the information of the others' utility is unknown, nobody can make a rational decision. In this case, some modification will be required.

Second, in this model, the private goods market and public goods market are completely divided, and the budget constraint of each individual is also divided into two parts by the budget system, one of which is to purchase private goods and the other of which is to contribute to public goods. These two parts are completely divided, so that free rider problems cannot occur. The private goods market is perfectly competitive, the honest demand revelation[xi] is conducted by vote in the public goods market. As a result of these completely divided markets, the allocation of resources becomes efficient.

The system in this model is, of course, a theoretical one. Someone may think the system is impossible to work and it is not worth considering. However, the similar systems have been adopted by some companies to supply fringe-benefits. So called cafeteria plan is a system that companies show the menu of fringe-benefits and employees select some from the menu according to their given points. Decentralized system can be possible at least in some categories of public goods. The system presented in this paper is an attempt to solve the serious political problems that tend to be considered impossible to settle.

Traditional works on the optimal provision of public goods such as Samuelson (1954,1955), Lindahl (1967), Bowen (1943), Clarke (1971), as Brown & Jackson (1992) pointed out, sought a quasi-competitive equilibrium depending on the initial distribution. And in this kind of equilibrium, for both private goods and public goods, marginal utility is required to equal marginal benefit. In this sense, these studies are in the tradition of the benefit principle. Only when the initial distribution is "fair," this kind of equilibrium is permitted by the norm of fairness. But, in the traditional studies of the optimal provision of public goods, there is no system that guarantees the initial distribution is fair[xii].

On the other hand, the allocation efficiency subjected to a budget system that is defined in this paper does not depend on the initial distribution, but depends on a state that is redistributed by a budget system. Thus, in this system, there is a possibility that the fairness is satisfied by redistribution[xiii]. There is a clear contrast with Lindahl's equilibrium that determines the tax depending on the initial distribution that is not guaranteed to be fair.

In this decentralized decision making system, one's tax payment can be decided independently from one's benefit. The amount of tax payment can be decided under the ability to pay principle. In this sense, this system can be one of the optimal public goods provision systems based on the ability to pay principle.

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i Samuelson, P. A. (1954,1955)

ii In the study of demand revelation, since Clarke (1971) created a system that lead people to the honest demand revelation and Tideman & Tullock (1976) determined its importance, many have tried to find a more useful system. Groves & Ledyard (1977) figured out a solution consistent with a Lindahl's equilibrium and balanced budget in a general equilibrium model. But it was too abstract to carry out.

iii Brennan & Buchanan (1980) analyzed problems about the overall level of the governmental activity and the disposition of resources that the government withdraws from the public sector independently. According to this classification, the analysis in this paper is about disposition.

iv In this model, public debt is neglected for simplicity.

v The distribution of the disposal right can be decided independently from or dependently on the tax structure.

vi a_i , b_i are the marginal utility of y_1 and y_2 , respectively, and are assumed to be constant, for simplicity.

vii In this case, it is indifferent for the i-th individual to choose any amount to contribute to each type of public goods. However, it is assumed that the i-th individual contributes the entire amount to y_1 (y_2), if there are some who want y_1 (y_2) and there is no one who wants y_2 (y_1). But, if both types of individuals want y_1 or y_2, the amount the i-th individual chooses is thought to be between all and nil.

viii The amount of tax levied on each individual is treated as given, and as a result of this, the ratio of resources used for public goods and that of private goods is given in this model.

ix In the 1 person–1 vote system, it is possible to think the disposal rights are equally distributed to all persons. In this model, the amount of tax levied on the i-th individual is independent from the i-th individual's disposable amount, and the governmental activity is assumed to be run only by tax. But, in general, public debt can be considered, and it is possible to relate one's disposable amount and the amount of tax levied.

x For example, in Shibata & Shibata (1988) p.185, " It is doubtful for beneficiaries to express their marginal utilities about abstract public goods like national defense in money terms."

xi Because personal budget is completely divided into two parts, one is for private goods and the other is for public goods, free riding behavior cannot occur. So, the honest demand for public goods will be revealed.

xii Brown & Jackson (1992) discusses the relationship between the optimal provision of public goods and fairness, and pointed out that both Wicksell (1967) and Lindahl (1967) care about fairness of initial distribution.

xiii This is under the assumption that the budget system works well from the perspective of fairness. But, if politics do not work well, it is a utopian idea to hope that politics will make a good budget system.