On Some Subclasses of the Class of All ω–Star–Free Regular Sets

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Contents

1. Introduction

The class of regular languages is defined to be the class of languages expressed by regular expressions. If we remove Kleene closure from the regular expression, we then get the concept of star-free regular expression. We call the language expressed by star-free regular expression the star-free regular set. We denote by **SF** the class of star-free regular sets. McNaughton and Papert [6] define not only **SF**, but also **NC** (noncounting), **LTO** (local testing and order), **GF** (group-free), **PF** (permutation-free) and so on, as subclasses of the class of regular sets, and conclude that all of these classes coincide with one another. Although in the literature [6] it is easy to prove $\mathbf{SF} \subseteq \mathbf{NC}$, the reverse inclusion, namely $\mathbf{NC} \subseteq \mathbf{SF}$, is not the case. Several classes mentioned above are arranged between **NC** and **SF**, and the proof of the inclusion $\mathbf{NC} \subseteq \mathbf{SF}$ is then achieved step by step in [6]. Indeed, it is laborious to prove $\mathbf{NC} \subseteq \mathbf{SF}$. On the other hand, Meyer [7] defines four classes of languages, i.e., **SF**, **NC**, **GF** and **RES** (= the class of languages accepted by finite automata whose semiautomata are cascade products of resets), and shows that the equality of these four classes holds true by means of Krohn-Rhodes' decomposition theorem. As to the proof of $\mathbf{NC} \subseteq \mathbf{SF}$, the method of Meyer [7] is easier than that of McNaughton and Papert [6].

By the way, the concept of ω-regular sets has already been obtained by previous works as an extension of regular sets to the ω-type version. In the same way, the class of ω-star-free regular sets, that is $\mathbf{SF}^\omega$, has been built up by Thomas [9]. As to the extension of star-free regular sets to the ω-type version, Ladner [4] proposes $\mathbf{LSF}^\omega$, which is the smallest class containing the empty set $\emptyset$ and is closed under union, complement, and product with a star-free regular set of words on the left. Thomas [9] proves the equality of $\mathbf{SF}^\omega$ and $\mathbf{LSF}^\omega$.

In this paper, we introduce four classes of ω-languages, that is, $\mathbf{MSF}^\omega$, $\mathbf{NC}^\omega$, $\mathbf{GF}^\omega$ and $\mathbf{RES}^\omega$. These are ω-type versions of **SF**, **NC**, **GF** and **RES** which have been introduced by Meyer [7]. Moreover, we will show that $\mathbf{MSF}^\omega = \mathbf{NC}^\omega = \mathbf{GF}^\omega = \mathbf{RES}^\omega$.

2. Symbols and Preliminaries

Our notation follows Meyer [7]. Let f and g be functions from set S to S. We denote a function value $f(s)$ by sf and define $s(f \circ g) = g(f(s))$.

We introduce a semiautomaton $A = < Q^A, \Sigma^A, M^A >$, where Q^A is a finite set of states, Σ^A is a finite set of inputs,and M^A is a set of mappings $M_\sigma^A : Q^A \to Q^A$ defined for each $\sigma \in \Sigma^A$. We abbreviate M_σ^A as σ^A. For each $x \in (\Sigma^A)^*$, we define a mapping $x^A : Q^A \to Q^A$ as follows. Λ^A is the identity mapping over Q^A. If $x = y\sigma$ for $y \in (\Sigma^A)^*$, $\sigma \in \Sigma^A$, then $x^A = y^A \circ \sigma^A$, where $\circ$ expresses the composition of mappings. Therefore a mapping x^A corresponds to each $x \in (\Sigma^A)^*$. As x runs over $(\Sigma^A)^*$, we obtain a set G^A that consists of the corresponding x^A, and the set G^A forms a semigroup under the composition $\circ$.

Adding an initial state $s_0^A \in Q^A$ and a final state set $F^A \subseteq Q^A$ to a semiautomaton A, we obtain a deterministic finite automaton $\hat{A} = < Q^A, \Sigma^A, s_0^A, M^A, F^A >$ whose semiautomaton is A.

We define several kinds of acceptances of the ω–sequence as follows. As a few special subsets of the set of all ω–sequences $(\Sigma^A)^\omega = \{\alpha \mid \alpha : \boldsymbol{\omega} \to \Sigma^A\}$,we consider the following ω–languages.

1) $O_1(\hat{A}) = \{\alpha \in (\Sigma^A)^\omega \mid \exists i \;\; s_0^A\alpha(0,i)^A \in F^A\}$
2) $O_2(\hat{A}) = \{\alpha \in (\Sigma^A)^\omega \mid \forall i \;\; s_0^A\alpha(0,i)^A \in F^A\}$
3) $O_3(\hat{A}) = \{\alpha \in (\Sigma^A)^\omega \mid \forall i \; \exists j \geq i \;\; s_0^A\alpha(0,j)^A \in F^A\}$
4) $O_4(\hat{A}) = \{\alpha \in (\Sigma^A)^\omega \mid \exists i \; \forall j \geq i \;\; s_0^A\alpha(0,j)^A \in F^A\}$

We say L is (1, 1', 2, 2')–accepted by $\hat{A}$ if $L = O_i(\hat{A})$ $(i = 1,2,3,4)$, and let $\mathbf{O}_i (i = 1,2,3,4)$ be the class of ω–languages expressed by $O_i(\hat{A})$ $(i = 1,2,3,4)$.

3. ω–Star–Free Sets and ω–Noncounting Sets

Definition 3.1 Let Σ be a finite set of letters.

(i) If $\sigma \in \Sigma$, then $\{\sigma\}$ is a star–free set over Σ.

(ii) If $U, V \subseteq \Sigma^*$ are star–free sets over Σ, then $U \cup V$, $\bar{U}$,and UV are also star–free sets over Σ.

(iii) A set is star–free over Σ only if that is required by (i) (ii).

The set of all star-free regular sets over Σ is denoted by $\mathbf{SF}_\Sigma$. If the context is clear, we omit Σ. Let $\mathfrak{R}$ denote the set of all regular languages. Clearly, $\mathbf{SF} \subseteq \mathfrak{R}$.

Definition 3.2 For $W \subseteq \Sigma^*$, we define

$$\lim W = \{x \in \Sigma^\omega \mid \forall i\ \exists j \geq i \ \ x(0,j) \in W\}.$$

$X \subseteq \Sigma^\omega$ is an *ω–star-free regular set iff* there are an integer $m \geq 1$ and $U_1, \cdots, U_m, V_1, \cdots, V_m \in \mathbf{SF}$ such that

$$X = \bigcup_{i=1}^{m} U_i \lim V_i.$$

We denote by $\mathbf{SF}^\omega_\Sigma$ the set of all ω–star–free regular sets over Σ.

Definition 3.3 Let $\mathbf{LSF}^\omega_\Sigma$ be the class defined as the closure of the empty set of ω–sequences over Σ under the operations of union, complement and concatenation with a star–free set of words on the left.

Theorem 3.4 (Thomas [9])
$\mathbf{SF}^\omega = \mathbf{LSF}^\omega$

Definition 3.5 A regular set $A \subseteq \Sigma^*$ is said to be *non-counting* if there exists an integer $k \geq 0$ such that for any $x, y, z \in \Sigma^*$ $\ xy^kz \in A$ *iff* $xy^{k+1}z \in A$.
We denote by $\mathbf{NC}$ the set of all noncounting regular sets.

Theorem 3.6 (Meyer [7])
$\mathbf{SF} \subseteq \mathbf{NC}$

Definition 3.7 An ω–language $U \subseteq \Sigma^\omega$ is said to be *ω – –noncounting* if there exists a noncounting regular set V such that $U = \lim V$. We denote by $\mathbf{NC}^\omega$ the set of all ω–noncounting sets.
We can obtain an ω–language $\lim V$ for each $V \subseteq \Sigma^*$ and we study here this lim–operator.

Theorem 3.8 (Choueka [1])
$U \in \mathbf{O}_3$ *iff* $U = \lim V$ *for some* $V \in \mathfrak{R}$

From this theorem, it is evident that $\mathbf{NC}^\omega \subseteq \mathbf{O}_3$.

Theorem 3.9 *For any regular set V, there exist an integer $m \geq 1$ and regular sets $A_1, \cdots, A_m, B_1, \cdots, B_m$ such that* $\lim V = \bigcup_{i=1}^{m} A_i \lim B_i$.

Proof. Assume that V is accepted by a deterministic finite automaton $\hat{A} = < Q, \Sigma, s_0, M, F >$, where $F = \{t_1, \cdots, t_m\}$. For $i = 1, 2, \cdots, m$, let $\hat{A}_i =< Q, \Sigma, s_0, M, t_i >, \hat{B}_i =< Q, \Sigma, t_i, M, F >$ and define $A_i = T(\hat{A}_i)$, $B_i = T(\hat{B}_i)$. Thereupon each A_i, B_i is a regular set and we will show the equality $\lim V = \bigcup_{i=1}^{m} A_i \lim B_i$.

If an $x \in \lim V$ is arbitrarily fixed, we can find $x_1, x_2, x_3, \cdots$ so that $x = x_1 x_2 x_3 \cdots$ and $s_0 x_1 \in F, s_0 x_1 x_2 \in F, s_0 x_1 x_2 x_3 \in F, \cdots$ and so on. Therefore $s_0 x_1 = t_i$ for some $t_i \in F$ $(i \leq m)$. So $x_1 \in A_i$. Since $t_i x_2 \in F, t_i x_2 x_3 \in F, \cdots$ we conclude $x_2 x_3 x_4 \cdots \in \lim B_i$. Hence, $x \in A_i \lim B_i$ for some $i \leq m$.

Conversely, suppose $x \in A_i \lim B_i$ for some $i \leq m$. Then $x = yz$ for some $y \in A_i$ and $z \in \lim B_i$. Consequently, $y \in V$. From $z \in \lim B_i$ we can decompose z as $z = z_1 z_2 z_3 \cdots$ and $t_i z_1 \in F, t_i z_1 z_2 \in F, t_i z_1 z_2 z_3 \in F, \cdots$. Since $y \in A_i$ we have $s_0 y = t_i$. Thus $s_0 y z_1 \in F, s_0 y z_1 z_2 \in F, s_0 y z_1 z_2 z_3 \in F, \cdots$ and so on. Hence, $y \in V, y z_1 \in V, y z_1 z_2 \in V; y z_1 z_2 z_3 \in V \cdots$. Therefore $x = yz \in \lim V$. The above proves that $\lim V = \bigcup_{i=1}^{m} A_i \lim B_i$. ∎

When $V \subseteq \Sigma^*$ is a star–free regular set, since $\{\Lambda\} \in \mathbf{NC} = \mathbf{GF} = \mathbf{SF}$, $\lim V = \{\Lambda\} \lim V$ becomes an ω–star–free regular set. In the same way, when V is a regular set, by Theorem 3.9, $\lim V$ is also an ω–regular set.

Lemma 3.10

$$\lim(A \cup B) = \lim A \cup \lim B$$

Proof. Trivial. ∎

Definition 3.11 $A \subseteq \Sigma^*$ is a *minimal set* iff A satisfies the following:

$$A \cap A\Sigma^+ = \emptyset.$$

Definition 3.12 $X \subseteq \Sigma^\omega$ is an *ω–minimal star-free set* iff there exist $m \geq 1$ and star–free regular sets $U_1, \cdots, U_m$, $V_1, \cdots, V_m$ such that

$$X = \bigcup_{i=1}^{m} U_i \lim V_i,$$

where each U_i is a minimal set.

We denote by $\mathbf{MSF}^\omega$ the set of all ω–minimal star–free sets.

Lemma 3.13 (Eilenberg [2]) *If a set A is minimal, then for all $B \subseteq \Sigma^*$,*

$$A \lim B = \lim(AB).$$

Proof. We will prove first $A \lim B \subseteq \lim(AB)$. If an ω–sequence $x \in A \lim B$ is arbitrarily fixed, we can decompose x as follows: $x = x_0x_1x_2x_3\cdots$, where $x_0 \in A$ and $x_1 \in B, x_1x_2 \in B, x_1x_2x_3 \in B, \cdots$. Therefore $x_0x_1 \in AB, x_0x_1x_2 \in AB, x_0x_1x_2x_3 \in AB, \cdots$. Consequently, we obtain $x \in \lim(AB)$.

Conversely, if $x \in \lim(AB)$, we can decompose $x = x_0x_1x_2\cdots$ so that $x_0 \in AB, x_0x_1 \in AB, x_0x_1x_2 \in AB, \cdots$. Thus $x_0 = yz$ for some $y \in A, z \in B$. Hence, $yz \in AB, yzx_1 \in AB, yzx_1x_2 \in AB, \cdots$. Since A is minimal and y is in A, we obtain $z \in B, zx_1 \in B, zx_1x_2 \in B, \cdots$ and so on. Therefore $zx_1x_2x_3\cdots \in \lim B$. So $x = yzx_1x_2x_3\cdots \in A\lim B$. The above proves that $A \lim B = \lim(AB)$. ∎

Theorem 3.14 $\mathbf{MSF}^\omega \subseteq \mathbf{NC}^\omega$

Proof. Let $U \subseteq \Sigma^\omega$ be an ω–minimal star–free set. Then, by Theorem 3.6, Lemma 3.10 and Lemma 3.13, there are an integer $m \geq 1$ and minimal noncounting regular sets $A_1, \cdots, A_m$ and noncounting regular sets $B_1, \cdots, B_m$ such that $U = \bigcup_{i=1}^m A_i \lim B_i = \bigcup_{i=1}^m \lim(A_iB_i) = \lim(\bigcup_{i=1}^m A_iB_i)$, where $\bigcup_{i=1}^m A_iB_i$ is a regular set.

Let $k_1, \cdots, k_m$, $l_1, \cdots, l_m$ be the integers determined from noncounting regular sets $A_1, \cdots, A_m$, $B_1, \cdots, B_m$, respectively, and define $k = \max\{k_1, \cdots, k_m\}$ and $l = \max\{l_1, \cdots, l_m\}$. Then we claim that for every $x, y, z \in \Sigma^*$ $xy^{k+l+1}z \in \bigcup_{i=1}^m A_iB_i$ *iff* $xy^{k+l+2}z \in \bigcup_{i=1}^m A_iB_i$.

We prove the 'only if' part because the 'if' part is similar to the reverse proof. Assume that $xy^{k+l+1}z \in A_iB_i$ for some $i \leq m$. Hence, $xy^{k+l+1}z = uv$ for some $u \in A_i, v \in B_i$. We must study two cases.

1) In the case of $|x| + k|y| \geq |u|$.

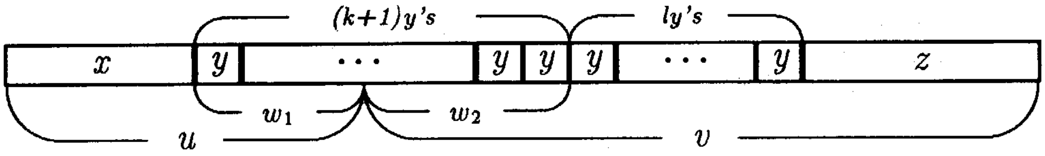

Figure 1.

When $|x| + k|y| \geq |u|$, then there exist $w_1, w_2 \in \Sigma^*$ such that $u = xw_1$ and $v = w_2y^lz$ and $w_1w_2 = y^{k+1}$. Since B_i is a noncounting set, from the fact that $v = w_2y^lz = w_2y^{l-l_i}y^{l_i}z \in B_i$, we obtain $w_2y^{l-l_i}y^{l_i+1}z = w_2y^{l+1}z \in B_i$. Therefore $xw_1w_2y^{l+1}z = xy^{k+l+2}z \in A_iB_i$.

2) In the case of $|x| + k|y| < |u|$.

When $|x|+k|y| < |u|$, we also arrive at the same conclusion using the fact that A_i is a noncounting set.

As for the inverse direction, we can perform the proof similarly. Since $\bigcup_{i=1}^m A_iB_i$ is a noncounting regular set, it follows that $U \in$ $\mathbf{NC}^\omega$. ∎

4. ω–Group–Free Sets

Definition 4.1 Let V be a subset of Σ. For $w, y \in \Sigma^*$ we define an equivalence relation $\equiv \pmod V$ as follows:

$w \equiv y \pmod V$ if for any $x, z \in \Sigma^*$

$$xwz \in V \quad \textit{iff} \quad xyz \in V.$$

Definition 4.2

(i) A *subgroup* of a semigroup S is a subset of S whose elements form a group under operation in S.

(ii) A semigroup S is *group-free* if and only if S has no subgroup except the trivial group with only the identity.

(iii) A semiautomaton A is *group-free* if and only if the semigroup G^A of A is group-free.

(iv)An ω–regular set $U \subseteq \Sigma^\omega$ is *group–free* if and only if there is a finite automaton $\hat{A}$ 2-accepting U such that the semiautomaton A of $\hat{A}$ is group–free.

We denote by $\mathbf{GF}^\omega$ the set of all group–free ω–regular sets. It is obvious that $\mathbf{GF}^\omega \subseteq \mathbf{O_3}$.

Lemma 4.3† (Nerode and Myhill's Theorem)
Let $V \subseteq \Sigma^*$ be a regular set. Then there is a reduced finite automaton $\hat{A}$ such that if $x \equiv y \pmod V$, then $x^A = y^A$ and $V = T(\hat{A})$.

Theorem 4.4 $\mathbf{NC}^\omega \subseteq \mathbf{GF}^\omega$

Proof. If $U \in \mathbf{NC}^\omega$, then there exists a regular set V such that $U = \lim V$, satisfying the following:
$\exists k \geq 0 \ \forall x, y, z \in \Sigma^*$

$$xy^k z \in V \qquad \textit{iff} \qquad xy^{k+1} z \in V.$$

Therefore $y^k \equiv y^{k+1} \pmod V$ for every $y \in \Sigma^*$. By Lemma 4.3, there is a finite automaton $\hat{A}$ such that $y^k = y^{k+1}$ and $U = \lim T(\hat{A})$.

Let G^A be a semigroup of A and suppose G to be any subgroup of G^A. Since the assertion $\exists k \geq 0 \ \forall y \in G^A \quad y^k = y^{k+1}$ holds, $e = y^k(y^{-1})^k = y^{k+1}(y^{-1})^k = y$ for every $y \in G \subseteq G^A$. Consequently, $G = \{e\}$. Namely, G^A is group-free. Hence, the semiautomaton A of $\hat{A}$ is group-free. Therefore $U \in \mathbf{GF}^\omega$. ∎

5. The Cascade Product of Reset Automata

Definition 5.1 Let A and B be semiautomata and consider a mapping $\Omega : Q^A \times \Sigma^A \rightarrow \Sigma^B$. *The cascade product of A and B* is the semiautomaton $C = < Q^A \times Q^B, \Sigma^A, M^C >$, where M^C is a set of mappings σ^C defined as follows: For $\sigma \in \Sigma^A$ and for any $s \in Q^A$, $t \in Q^B$ $\quad < s,t > \sigma^C = < s\sigma^A, t(< s, \sigma > \Omega)^B >$. A

†Dr.Tetsuo Moriya pointed out my mistake about this lemma in the preceding version of this paper in [8].

cascade product of three or more automata is defined by association to the left(cf. [3]).

Definition 5.2 A semiautomaton R is a *reset automaton* if $Q^R = \{1,2\}$ and $\Sigma^R = \Sigma_1^R \cup \Sigma_2^R \cup \Sigma_I^R$, where Σ_1^R, Σ_2^R, and Σ_I^R are mutually exclusive input alphabets defined as follows:

$\Sigma_1^R = \{\sigma |\ \text{range}(\ \sigma^R\ ) = \{1\}\}$,
$\Sigma_2^R = \{\sigma |\ \text{range}(\ \sigma^R\ ) = \{2\}\}$,
$\Sigma_I^R = \{\sigma |\ \sigma^R$ is the identity mapping on $Q^R\ \}$.

Definition 5.3 Let A and B be two semiautomata. B is a *homomorphic image* of A if and only if $\Sigma^A = \Sigma^B$ and there is an onto mapping $\eta : Q^A \to Q^B$ such that $\eta \circ \sigma^B = \sigma^A \circ \eta$ for each $\sigma \in \Sigma^A$. Such a mapping η is called a homomorphism of A onto B.

Definition 5.4 Let A and B be semiautomata. B is a *subsemiautomaton* of A if and only if the following three conditions hold:

(i) $Q^B \subseteq Q^A$,
(ii) $\Sigma^B \subseteq \Sigma^A$,
(iii)For each $\sigma \in \Sigma^B$, the mapping σ^B is a restriction of σ^A to Q^B.

Theorem 5.5 (Krohn–Rhodes' Decomposition Theorem)
Every semiautomaton A is a homomorphic image of a subsemiautomaton of a cascade product of semiautomata $A_1, A_2, \cdots, A_n$ such that for $1 \leq i \leq n$, A_i is a reset automaton, or else G^{A_i} is a nontrivial homomorphic image of a subgroup of G^A.

Proof. See Ginzburg[3]. ∎

Definition 5.6 We define the following set.
$\mathbf{RES}^\omega = \{U \subseteq \Sigma^\omega |\ U$ is 2-accepted by a finite automaton whose semiautomaton is a cascade product of reset automata.$\}$

Theorem 5.7 $\mathbf{GF}^\omega \subseteq \mathbf{RES}^\omega$

Proof. If $U \in \mathbf{GF}^\omega$, then there exists a finite automaton $\hat{A} = < Q^A, \Sigma^A, s_0^A, M^A, F^A >$ 2-accepting U, where G^A is group–free. So when we apply Theorem 5.5 to the semiautomaton A of $\hat{A}$, it appears that A is a homomorphic image of a subsemiautomaton of a cascade product C of reset automata $A_1, \cdots, A_n$. We may assume that $\Sigma^A = \Sigma^C$ because the subsemiautomaton of C obtained by restricting Σ^C to Σ^A is also a cascade product of reset automata and A is a homomorphic image of the restricted subsemiautomaton of C.

Fix a state $s_0^C \in \eta^{-1}(s_0^A)$ arbitrarily, and then define a set

$$F^C = \{q \in Q^C \mid \eta(q) \in F^A\}$$

to make a finite automaton $\hat{C} = < Q^C, \Sigma^C, s_0^C, M^C, F^C >$.

Then for any $x \in (\Sigma^A)^\omega = (\Sigma^C)^\omega$,

$$\begin{aligned}
x \in U \quad & \textit{iff} \quad \forall i\ \exists j \geq i \quad s_0^A x(0,j)^A \in F^A \\
& \textit{iff} \quad \forall i\ \exists j \geq i \quad \eta(s_0^C) x(0,j)^A \in F^A \\
& \textit{iff} \quad \forall i\ \exists j \geq i \quad s_0^C(\eta \circ x(0,j)^A) \in F^A \\
& \textit{iff} \quad \forall i\ \exists j \geq i \quad s_0^C(x(0,j)^C \circ \eta) \in F^A \\
& \textit{iff} \quad \forall i\ \exists j \geq i \quad \eta(x(0,j)^C(s_0^C)) \in F^A \\
& \textit{iff} \quad \forall i\ \exists j \geq i \quad x(0,j)^C(s_0^C) \in F^C \\
& \textit{iff} \quad \forall i\ \exists j \geq i \quad s_0^C x(0,j)^C \in F^C.
\end{aligned}$$

Consequently, U is 2-accepted by $\hat{C}$. So $U \in \mathbf{RES}^\omega$ since the semiautomaton of $\hat{C}$ is a cascade product of reset automata. ∎

Theorem 5.8 $\mathbf{RES}^\omega \subseteq \mathbf{MSF}^\omega$

Proof. Suppose that $U \subseteq \Sigma^\omega$ is 2-accepted by a finite automaton $\hat{A}$ whose semiautomaton A is a cascade product of reset automata. According to Meyer[7], $T(\hat{A})$ is a star-free regular set. Since $\{\Lambda\}$ is minimal star-free, $U = \{\Lambda\} \lim T(\hat{A}) \in \mathbf{MSF}^\omega$. ∎

From Theorem 3.14, Theorem 4.4, Theorem 5.7 and Theorem 5.8, we obtain the following main result.

Theorem 5.9 (Main Theorem)

$$\mathbf{MSF}^\omega = \mathbf{NC}^\omega = \mathbf{GF}^\omega = \mathbf{RES}^\omega$$

6. A Related Result

From definitions, it is obvious that $\mathbf{MSF}^{\omega} \subseteq \mathbf{SF}^{\omega}$. Does this inclusion hold strictly? The answer is " YES ".

Theorem 6.1 $\mathbf{MSF}^{\omega} \subset \mathbf{SF}^{\omega}$

Proof. Since $\mathbf{MSF}^{\omega} \subseteq \mathbf{O_3}$, $\mathbf{SF}^{\omega} - \mathbf{O_3} \subseteq \mathbf{SF}^{\omega} - \mathbf{MSF}^{\omega}$. Therefore, it suffices to show the existence of X such that $X \in \mathbf{SF}^{\omega} - \mathbf{O_3}$.

Suppose $\Sigma = \{0, 1\}$ and let $X = \Sigma^* 0^{\omega}$. Then $X = \Sigma^* \lim 0^* \in \mathbf{SF}^{\omega}$. On the other hand, it is known that X does not belong to $\mathbf{O_3}$ from Landweber[5]. ∎

7. Conclusions

We illustrate the above-mentioned results as follows.

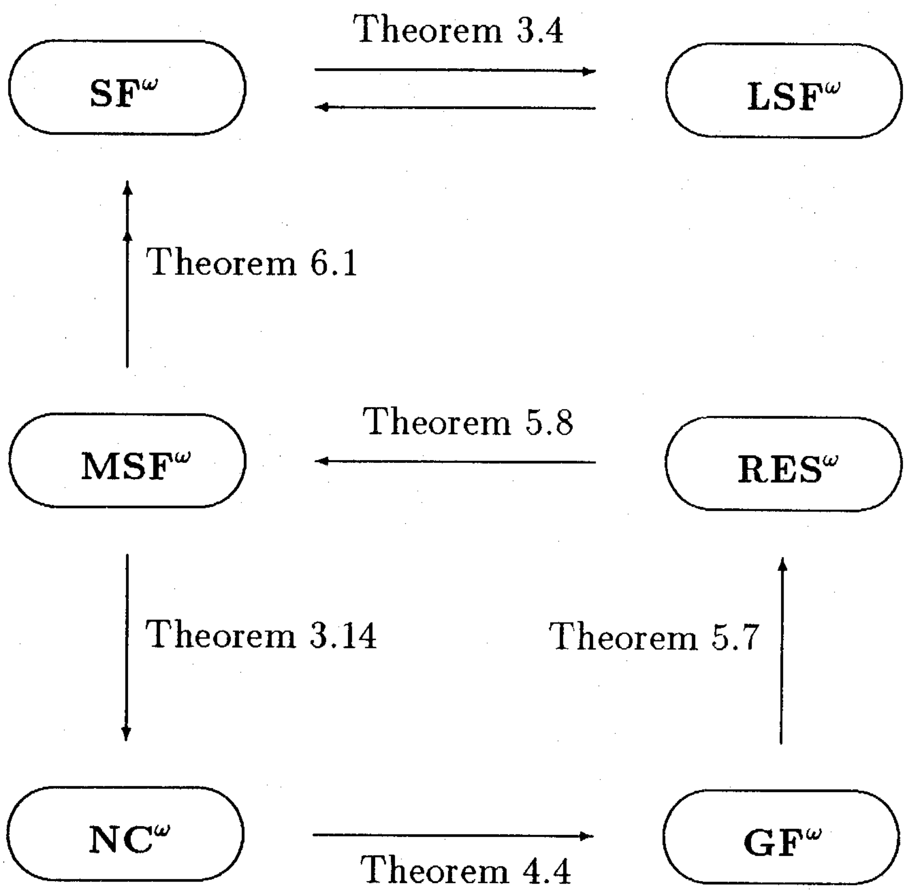

Figure 2.

Hereafter, we will have many kinds of classes of ω–languages which originated from McNaughton and Papert[6]. For example, $\mathbf{LTO}^\omega, \mathbf{PF}^\omega$ and so on, and we will investigate the relation between these classes and the ones obtained in this paper.

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